Exam Subatomic Physics

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Before you start, read the following:

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- *Good luck!*

1 Allowed and Forbidden Processes (15 Points)

Examine the following processes, and state for each one whether it is *possible* or *impossible*, according to the Standard Model. In the former case, state which interactions is responsible – strong, electromagnetic or weak; in the latter case, cite a conservation law that prevents it from occurring. When unambiguous, the charge is not indicated, thus γ, Λ, and n are neutral; p is positive, e is negative, etc. (1 point per process)

(a) $p + \overline{p} \to \pi^+ + \pi^0$

(b) $\eta \to \gamma + \gamma$

(c) $\Sigma^0 \to \Lambda + \pi^0$

(d) $\Sigma^- \to n + \pi^-$

(e) $e^+ + e^- \to \mu^+ + \mu^-$

(f) $\mu^- \to e^- + \overline{\nu}_e$

(g) $\Delta^+ \to p + \pi^0$

(h) $\overline{\nu}_e + p \to n + e^+$

(i) $p \to e^+ + \gamma$

(j) $p + p \to p + p + p + \overline{p}$

(k) $n + \overline{n} \to \pi^+ + \pi^- + \pi^0$

(l) $\pi^+ + n \to \pi^- + p$

(m) $K^- \to \pi^- + \pi^0$

(n) $\Sigma^+ + n \to \Sigma^- + p$

(o) $\Xi^0 \to p + \pi^-$

2 Nuclear Masses (10 Points)

Consider the Bethe-Weizsäcker nuclear mass formula.

(a) Explain the origin of each of the terms (3 points).

(b) Consider a nucleus with atomic weight A. Show that the number of protons Z of the most stable nucleus is about $Z \simeq A/2$ (4 points).

(c) Find the atomic weight A of the most strongly bound nucleus using the result above (3 points).

3 Fermi's Golden Rule (15 Points)

Of the three charged leptons, *i.e.* the electron, muon and tau, only the electron is stable. The muon and tau can both decay into an electron and two neutrinos. The transition rates for these processes are given by

$$\Gamma(\mu^- \to e^- + \overline{\nu}_e + \nu_\mu) = K G_F^2 m_\mu^5$$

and

$$\Gamma(\tau^- \to e^- + \overline{\nu}_e + \nu_\tau) = K' G_F^2 m_\tau^5.$$

Here m_μ and m_τ are the particle masses and K, K' and G_F are constants.

(a) Which interaction is responsible for these decays? (3 points)

(b) What is the role of G_F? (3 points)

(c) What can you say about the difference between K and K' (if any)? (3 points)

(d) What is the origin of the m_l^5 factor?(3 points)

(e) Experimentally it is found that the ratio of the lifetimes of the τ and μ are given by

$$\frac{\tau_\mu}{\tau_\tau}\left(\frac{m_\mu}{m_\tau}\right)^5 \simeq 5.$$

What is the reason for this? (3 points)

4 Reactions (15 Points)

A group of physicists wants to perform an experiment on $^{82}_{37}$Rb, which has a halflife of 1.3 min. For a steady supply of $^{82}_{37}$Rb, they bombard~~s~~ $^{82}_{36}$Kr with a beam of α particles to produce $^{82}_{38}$Sr via the reaction $\alpha + ^{82}_{36}\mathrm{Kr} \to ^{82}_{38}\mathrm{Sr} + X$. These strontium nuclei subsequently decay to rubidium with a halflife of 25 days.

(a) What is X? (3 points)

(b) The cross section for the $^{82}_{38}$Sr production reaction is about 500 mb. If the target thickness is 1.64 mg/cm^2 and the beam current 10^{10} α's per second, how many strontium nuclei are then produced per second? (3 points)

(c) Which interaction is responsible for the transmutation of strontium to rubidium? Besides the rubidium nucleus, which other particle(s) is(are) produced when strontium decays? (3 points)

(d) After irradiating the Kr-target for a day, how many Rb decays will be observed per minute one week after the irradiation? And one week after that? (3 points)

(e) The decay $^{82}_{37}\mathrm{Rb}$ is mostly to the $J^P = 0^+$ ground state of $^{82}_{36}\mathrm{Kr}$, which is stable. In about 23% of the cases the krypton nucleus is left in a $J^P = 2^+$ excited state with an excitation energy of $E_x = 776\,\mathrm{keV}$. Which interaction is responsible for de-excitation? (3 points)

5 Neutrinos (15 Points)

From 1999 to 2006, the Sudbury Neutrino Observatory (SNO) used a 6 m radius sphere filled with heavy water, *i.e.* water with the hydrogen atoms replaced by deuterium atoms: D_2O, surrounded by photo-detectors to make measurements on solar neutrinos. Three reactions were studied:

$$\begin{array}{lcll} \nu + d & \rightarrow & e + p + p & \text{(I.)} \\ \nu + d & \rightarrow & \nu + p + n & \text{(II.)} \\ \nu + e & \rightarrow & \nu + e & \text{(III.)} \end{array}$$

(a) For each reaction I. – III., indicate how it's occurrence could be detected. (3 points)

(b) For each reaction I. – III., draw the Feynman diagram(s) of the dominant process(es) and indicate which boson and which (anti-)neutrino(s) is(are) involved. (3 points)

(c) Which type(s) of neutrinos are produced in the sun? (3 points)

(d) Neutrinos are known to exhibit flavor oscillations. Which *two* ingredients are necessary for such oscillations to occur? What is the role of the PMNS matrix? (3 points)

(e) Assume that the cross section for each of the processes for which you drew the Feynman diagram in (c) is $10^{-20}\,\mathrm{fm}^2$. The total solar neutrino flux on earth is $6 \times 10^{14}\,\mathrm{m}^{-2}\mathrm{s}^{-1}$. How many events of type I. – III. are detected if *no* flavor oscillations occur? And how many if 1/3 of the neutrinos on earth were ν_e's? *Caveat*: consider the mass difference between D and H and the composition of the water molecule. (3 points)

Constants

Speed of light	c	$2.998 \cdot 10^{8}$	m/s
Planck constant	h	$4.136 \cdot 10^{-24}$	GeV·s
	$\hbar = \frac{h}{2\pi}$	$6.582 \cdot 10^{-25}$	GeV/c
Electron charge	e	$1.602 \cdot 10^{-19}$	C
Electron mass	m_e	0.510998918(44)	MeV/c^2
Proton mass	m_p	938.272029(80)	MeV/c^2
Neutron mass	m_n	939.565360(81)	MeV/c^2
Deuteron mass	m_d	1875.61282(16)	MeV/c^2
Alpha particle mass	m_α	3727.37917(32)	MeV/c^2
Electron neutrino mass	m_{ν_e}	< 2.2	eV/c^2
Muon mass	m_μ	105.658369(9)	MeV/c^2
Tau mass	m_τ	1776.84(17)	MeV/c^2
Charged pion mass	$m_{\pi^\pm}$	139.57018(35)	MeV/c^2
Neutral pion mass	m_{π^0}	134.9766(6)	MeV/c^2
$W^\pm$-boson mass	m_W	80.403(29)	MeV/c^2
Z^0-boson mass	m_W	91.1876(21)	MeV/c^2
Avogadro's number	N_A	$6.02214179(30) \cdot 10^{23}$	mol^{-1}

Nuclear Masses (Bethe-Weizsäcker)

$$M(A,Z) = Nm_n + Zm_p - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

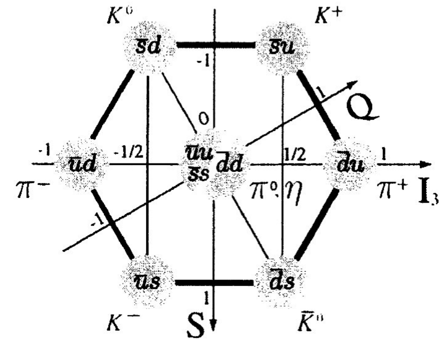
$$\begin{aligned}
a_v &= 15.67 \ \text{MeV}/c^2 \\
a_a &= 17.23 \ \text{MeV}/c^2 \\
a_c &= 0.714 \ \text{MeV}/c^2 \\
a_a &= 93.15 \ \text{MeV}/c^2 \\
\delta &= 0 \ \text{odd } A \\
&= -11.2 \ \text{MeV}/c^2, \quad Z \text{ and } A \text{ even} \\
&= +11.2 \ \text{MeV}/c^2, \quad Z \text{ and } A \text{ odd}
\end{aligned}$$

Conversion Factors

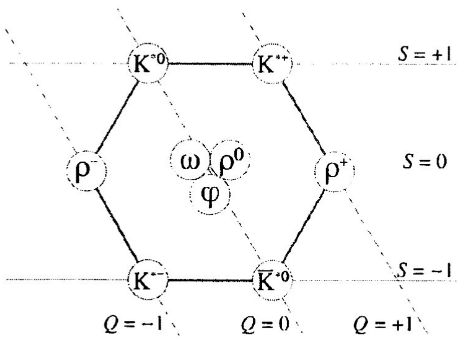
Electronvolt	eV	$1.60217653(14) \cdot 10^{-19}$	J
Tesla	T	$0.561 \cdot 1030$	MeV/c^2·C·s
kilogram	kg	$5.60958896(48) \cdot 10^{35}$	eV/c^2
barn	b	$1 \cdot 10^{-28}$	m^2

Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. Unless specifically stated, the final results are sufficient if given to 2 significant figures (2 leading digits).

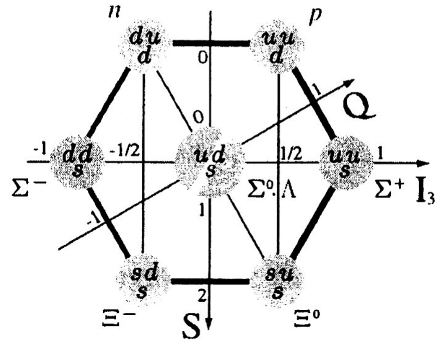
Baryon and Meson Composition



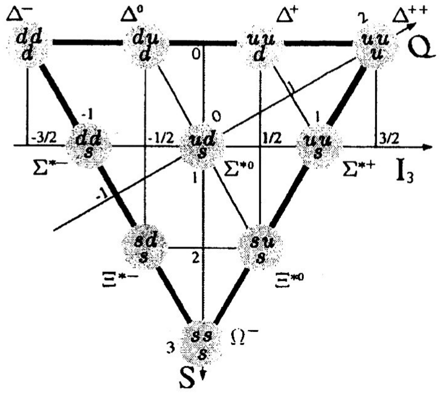
Spin-0 Mesons



Spin-1 Mesons



Spin-1/2 Baryons



Spin-3/2 Baryons